Modelling and Measurement of Electronically Tunable Photonic Crystals

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ABSTRACT
This paper presents details of the modelling that has been used to extract particle spacing from measured diffraction patterns in tunable photonic crystal structures. The photonic crystals are formed from colloidal suspensions of latex spheres with the application of rotating Alternating Current (AC) electric fields. The particle spacing can be rapidly changed by altering the amplitude of the AC field. Initially simple 1D diffraction grating theory is used to extract the particle spacing. This is followed by Finite Element (FE) modelling which shows good agreement with the simple theory. Full 3D modelling was investigated but only small number of spheres could be modelled.
Keywords : Photonic Crystal, diffraction grating, Finite Elements.

1. INTRODUCTION

The ability to electronically control the periodicity of 2D colloidal photonic crystals has recently been presented in [1]. This paper presents details of the modelling that has been used to interpret the measured results. In [1] it was shown how charged latex spheres can be formed into 2D periodic arrays by applying a rotating AC electric field. The AC field induces a dipole moment in the counter-ion layer and this then forms the particles into linear strings for linear fields and 2D Hexagonally Closed Packed (HCP) arrays for rotating fields. By changing the amplitude of the AC field the spacing between the particles can be rapidly changed. There are many important applications for such a system, including pixels in colour displays and tunable filters.

The spheres are dispersed in an aqueous solution and are positioned on a microscope glass slides and held in place by a thin glass cover slip. Four electrodes are positioned around a central observation area as shown in figure 1.

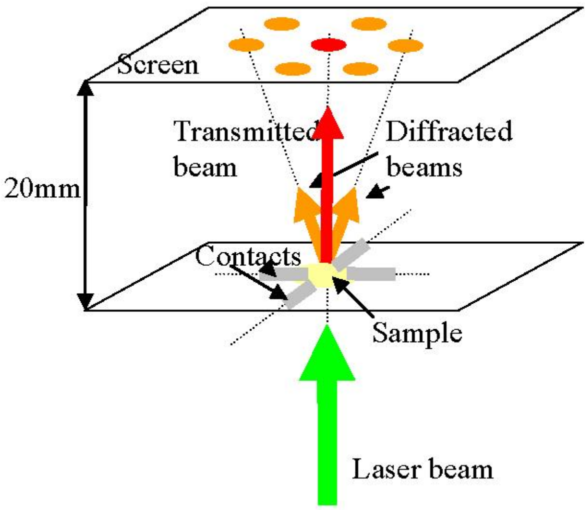

Figure 1. Green laser light illuminating observation area with diffraction patterns observed above the sample.

Linear field at any angle can be obtained by driving the two orthogonal pairs of electrodes at the same frequency and phase by adjusting the amplitude applied to the electrodes. Rotating fields can be obtained by driving the electrodes 90 degrees out of phase. Green laser light then illuminated the particles from below and the diffraction patterns were observed on a screen above the sample as shown in figure 1.

The particles are made from latex and have a diameter of 945nm and a refractive index of 1.6. They are dispersed in water which has a refractive index of 1.33 and the glass slide and cover slip have a refractive index of 1.5. Section 2 of this paper will briefly outline measured results which have been presented in more detail in [1]. Section 3 will then give a detailed account of the modelled results presented in [1] followed by 2D numerical modelling of 1D arrays of cylinders using the Finite Element method used to confirm the simple diffraction

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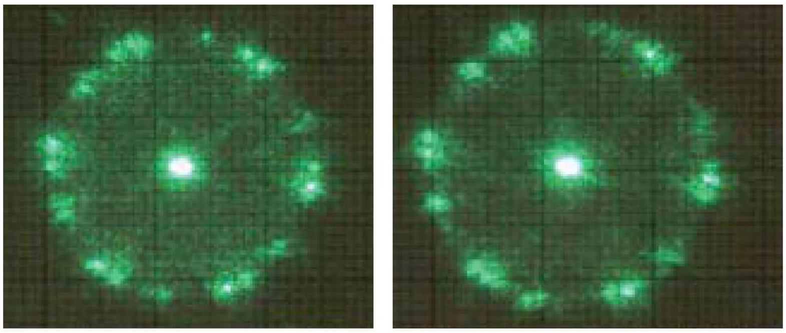
theory that has been used. More extensive full 3D modelling is now being carried out and these details will be the subject of future publications.

2. MEASURED RESULTS

When focused laser light illuminates a 2D HCP array of particles a six spot far field diffraction pattern will be observed. This is produced by interference from the three effective linear gratings produced by the hexagonal symmetry of the structure. Figure 2 shows the measured diffraction patterns at two different applied field strengths.



(a) (b)

Figure 2. Measured green laser diffraction patterns at normal incidence on 2D HCP rafts of particles at two different field strengths (a)20 kV/m (b)81 kV/m.

It can be seen that six spot patterns are not clearly observed since there are many rafts in the field of view each with random orientation. The radius of the pattern can still be used to extract particle spacing and section 3 describes the modelling required to extract this data. The change in radius was measured by using the graticule scale on the diffraction screen seen in the figure. Since the particles are quite large the particle spacing has been extracted directly from optical measurements as well. Figure 3 shows a graph of optically measured particle spacing vs. applied AC rotating field amplitude, this shows the wide tunability of the periodicity that has been achieved.

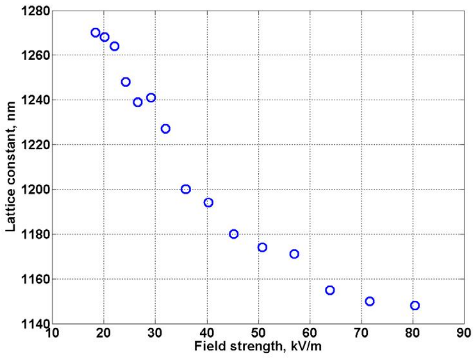


Figure 3. Optically measured particle spacing vs. applied AC field strength.

Having shown a summary of the measured results the paper will now go on to describe the different modelling approaches that have been taken to describe the optical performance of the system.

3. MODELLED RESULTS

A schematic view of the modelled diffraction grating is shown in Figure 4. Initially the simplest model for the 2D array of spheres is a 1D surface grating [2] which can be defined with von Laue relation and Snell's law as described in the equations below

$$a \cdot \sin(\theta_{diff}) = m\lambda / n_{av} \tag{1}$$

$$n_{av} \cdot \sin(\theta_{diff}) = n_1 \cdot \sin(\theta_1) = n_2 \cdot \sin(\theta_2) = n_3 \cdot \sin(\theta_3) \tag{2}$$

where $n_{av} = n_4 s_4 + n_1 s_3$, $s_4 = \left(\pi(d/2)^2\right)/(da)$, $s_3 = \left(da - \pi(d/2)^2\right)/(da)$, a is the lattice constant, θ_{diff} is the angle of the diffracted beam, m is the diffraction order, λ is the vacuum wavelength, n_{av} is the averaged refractive index [2], n_1 is the refractive index of the water, θ_1 is the angle of the diffracted beam in the water,

n_2 is the refractive index in the glass, θ_2 is the angle of the diffracted beam in the glass, n_3 is the refractive index of air, θ_3 is the angle of the diffracted beam in air, n_4 is the refractive index of the particles, s_4 is the filling fraction of cylinder in unit cell, s_3 is the filling fraction of water in unit cell, d is the cylinder diameter.
Substituting (2) into (1) we can find that the lattice spacing in the grating can be defined from the angle of the diffracted beam in air

$$a = \frac{m\lambda}{n_3 \sin(\theta_3)} \tag{3}$$

The measurement of the angle θ_3 is challenging task, but the position of the diffracted beams spots on the screen can be easily measured. Therefore we solve simple equation in order to find the angles of diffracted beams in the water or air

$$A = t_1 \tan(\theta_1) + t_2 \tan(\theta_2) + t_3 \tan(\theta_3) \tag{4}$$

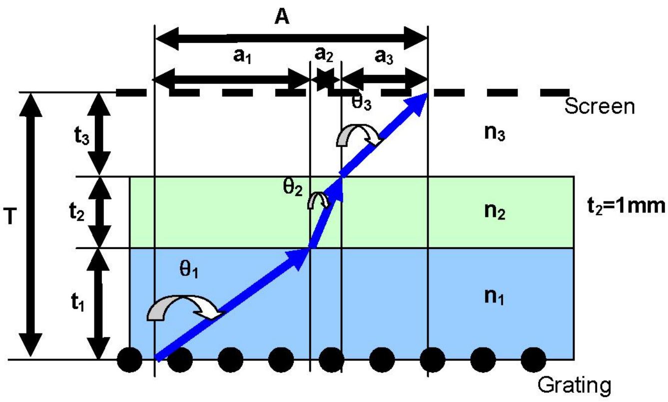


Figure 4. Schematic of the grating covered by water and glass layers, t3 = 20 mm, t2 = 1 mm and t1 = 380 nm.

The equations above where then used in combination with the measured diffraction data to predict the particle spacing. A comparison with optical measured particle spacing is shown in figure 5

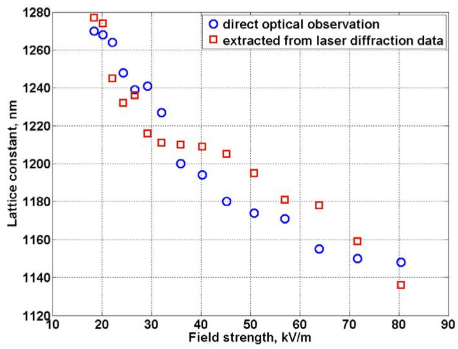
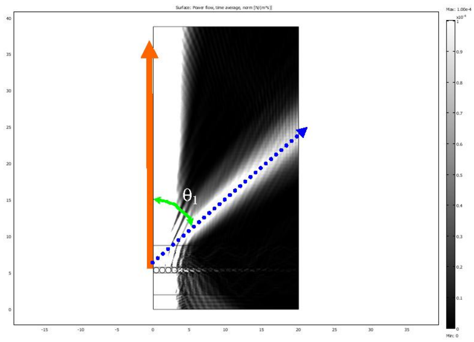
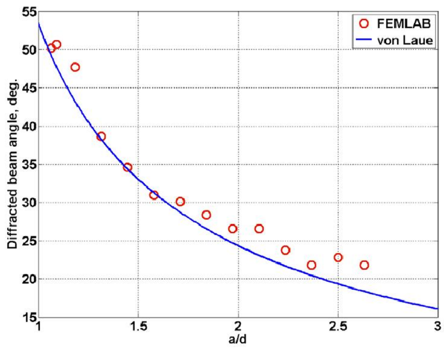


Figure 5. A comparison of optically measured particle spacing with that extracted using Von Laue theory d = 945nm.

It can be seen that good agreement has been obtained, the discrepancies are due to inaccuracies introduced in optical measurements of the particle spacing and in the measurements of the radius of the diffraction pattern.

In order to confirm the validity of the Von Laue approach and to move towards 3D modelling of the 2D HCP rafts a FE code was used from Comsol Multiphysics [3]. Initially a 2D model was constructed as shown in Fig. 6*a*. In this case a different particle diameter of 760 nm has been used, but it is straightforward to compare this with Von Laue since it is a closed form technique. Fig. 6*a* shows how a Gaussian beam is incident from beneath the grating and the diffracted beam is clearly observed. Careful meshing of the grating area is required in order to achieve converged results. Here the beam angle can be extracted directly and is plotted in Fig. 6*b* and compared with the Von Laue data. It can be seen that very good agreement has been achieved.





(a) (b)

Figure 6. (a) Power flow distribution of the beam propagating via a grating showing the diffracted beam angle, λ = 635 nm, n_1 = 1.33, n_2 = 1.5, n_3 = 1.3, n_4 = 1.6, d = 760 nm (b) First-order diffracted beam angle in air computed using von Laue relation (line) and Comsol Multiphysics package (circles) against normalised particle spacing.

The use of FE to model 2D arrays of spheres in full 3D was then investigated. However, it was found that even with a 64Bit Windows machine with 8 GB of RAM only very small number of spheres could be modelled and thus Finite Difference Time Domain Modelling was then pursued. These results will be the subject of future publications.

4. CONCLUSIONS

This paper has given details of the modelling used in [1] to extract particle spacing from measured diffraction data from 2D HCP rafts of spherical particles. The Von Laue relation has been used, but care must be taken to properly interpret the measured data to allow for the effect of Snell's law in the layers of water and glass above the particles. The Finite Element method was then used to confirm the Von Laue approach and to explore the modelling of 3D systems. It was found that only small number of spheres could be modelled using quite large amounts of RAM and it seems that techniques such as FDTD will be required to model such structures in full 3D.

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